May 2002

Bachelor of Computer Application (BCA) Examination

II Semester

Mathematics - II

Time 3 Hours] [Max. Marks 50

Note : *All questions are compulsory and carry equal marks.*

1. (a) Trace the curve : $y^2 = x^4 (a + x)$.

   (b) Test the convergency of following integral :

   $$\int_2^\infty \frac{dx}{\sqrt{x^2 - x - 1}}$$

OR

   (a) Trace the curve : $r^2 = a^2 \cos 2\theta$.

   (b) Test the convergency of the following integral :

   $$\int_\pi^\infty \frac{\sin^2 x\, dx}{x^2}$$

2. (a) Prove that $\Gamma(n)\, \Gamma\left(n + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2n-1}} \Gamma(2n)$

   (b) Prove that the perimeter of the Cardioide $r = a(1 + \cos\theta)$ is 8a.

OR

   (a) Evaluate $\int_0^1 \left(\log \frac{1}{y}\right)^{n-1} dy$.

   (b) Prove that the intrinsic equation of the curve $y = c \log \sec\left(\frac{x}{c}\right)$ is : $s = c \log (\sec \psi + \tan \psi)$

3. (a) Evaluate $\iiint_V (x + y + z)\, dx\, dy\, dz$ where the region V is bounded by the planes $x + y + z = (a > 0)$ and $x = 0, y = 0, z = 0$.

   (b) If $\vec{r}(t) = 2\hat{i} - \hat{j} + 3\hat{k}$ when $t = 2$

   $= 4\hat{i} + 2\hat{j} + 3\hat{k}$ when $t = 3$

evaluate $\int_{2}^{3} \vec{r} \cdot \frac{d\vec{r}}{dt} \, dt$

OR

(a) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{dxdydz}{(x+y+z+1)^2}$

(b) State the divergence theorem and verify it for $\vec{A} = 2x^2y\hat{i} - y^2\hat{j} + 4xz^2\hat{k}$ taken over the region in the first octant bounded by $y^2 + z^2 = 9$ and $z = 2$.

4. (a) Evaluate $\lim_{(x, y) \to (0, 0)} \frac{xy^3}{x^2 + y^6}$

(b) If $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$ prove that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u.$$

OR

(a) Examine the continuity of the function

$$f(x, y) = \frac{xy}{\sqrt{x^2 + y^2}} \quad \text{when } x \neq 0, y \neq 0$$
$$= 0 \quad \text{when } x = 0, y = 0.$$

(b) If $u = f\{(x - y); (y - z); (z - x)\}$

prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0.$

5. (a) Find the maximum value of $u = \sin x \sin y \sin (x + y)$.

(b) Test the convergency of the series whose n^{th} term is $\frac{\sqrt{n}}{n^2 + 1}$

OR

(a) Find the minimum value of $x^2 + y^2 + z^2$, given $ax + by + cz = p$.

(b) Test the convergency of the series whose n^{th} term is $\frac{n^{n^2}}{(1+n)^{n^2}}$

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