May 2003

Bachelor of Computer Application (BCA) Examination

II Semester

Mathematics - II

Time 3 Hours] [Max. Marks 40

***Note :** All questions are compulsory and carry equal marks.*

1. (a) Trace the curve $a^2y^2 = x^2(a^2 - x^2)$.

   (b) Test the convergence of the integral :

   $$\int_a^\infty e^{-x} \frac{\sin x}{x^2} dx \qquad \text{where } a > 0.$$

   (a) Trace the curve $r = a(1 + \cos\theta)$.

   (b) Examine the convergence of the integral :

   $$\int_0^1 \frac{dx}{x^{1/2}(1-x)^{1/3}}$$

2. (a) Prove that :

   $$\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)} \quad (m, n > 0).$$

   (b) Find the length of the arc of the curve $y = \log \frac{e^x - 1}{e^x + 1}$ form $x = 1$ to $x = 2$.

   OR

   (a) Prove that $\Gamma(n)\,\Gamma(1 - n) = \frac{\pi}{\sin n\pi}$, $0 < n < 1$.

   (b) Show that the intrinsic equation of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ is $s = 4a \sin\psi$

3. (a) Evaluate $\int_R\int (x^2 + y^2)\, dx\, dy$ where R is the region of integration bounded by

   $$x = 0,\ y = 0,\ x + y = 1.$$

   (b) Apply Gauss's divergence theorem to evaluate :

   $$\int_S\int [(x^3 - yz)\, dy\, dz - 2x^2y\, dz\, dx + 3dx\, dy]$$

over the surface of cube bounded by the co-ordinate planes and the planes x = y = z = a.

OR

(a) Evaluate $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z}\, dx\, dy\, dz$.

(b) Verify Stokes theorem for the function :

$$\vec{F} = (x^2 + y^2)\,\hat{i} - 2xy\,\hat{j}$$

taken round the rectangle bounded by $x = \pm a$ and $y = 0, y = b$.

4. (a) Investigate the continuity of the function :

$$f(x,y) = \begin{cases} \dfrac{xy^2}{x^2 + y^4}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases} \quad \text{at } (0, 0).$$

(b) If $u = \log(x^2 + y^2 + z^2)$ then prove that :

$$(x^2 + y^2 + z^2)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = 2.$$

OR

(a) If $u = \sin^{-1} \dfrac{x^3 - y^2}{x + y}$ then prove that $\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = z \tan u$.

(b) State and prove mean value theorm for a function of two variable.

5. (a) Discuss the maximum or minimum value of $u = xy + \dfrac{a^3}{x} + \dfrac{a^3}{y}$

(b) Test the convergence of the series :

$$1^2 + 2^2x + 3^2x^2 + 4^2x^3 + 5^2x^4 + \ldots,$$

where x is positive.

OR

(a) Find the minimum value of $x^2 + y^2 + z^2$, given $ax + by + cz = p$.

(b) Test the convergence of the series :

$$\frac{1}{1^p} + \frac{1}{3^p} + \frac{1}{5^p} +$$

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