May 2004

Bachelor of Computer Application (BCA) Examination

II Semester

Mathematics - II

Time 3 Hours] [Max. Marks 40

Note : *All questions are compulsory and carry equal marks.*

1. (a) Trace the curve $xy^2 = 4a^2(2a - x)$.

   (b) Test the convergence of the integral :

$$\int_2^{\infty} \frac{dx}{\sqrt{x^2 - 1}}$$

OR

   (a) Trace the curve $r^2 = a^2 \cos 2\theta$.

   (b) Test the convergence of the integral :

$$\int_0^1 \frac{dx}{x^{1/2}(1-x)^{1/3}}$$

2. (a) Prove that $\Gamma(n)\,\Gamma\left(n+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2n-1}}\,\Gamma(2n)$ where n is positive real number.

   (b) Prove that the arc length of the curve $y = \log \sec x$ from $x = 0$ to $x = \frac{\pi}{3}$ is $\log_e(2+\sqrt{3})$.

OR

   (a) Show that $\beta(m, n) = \int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} dx$.

   (b) Show that the intrinsic equation of the semi-cubical parabola $3ay^2 = 2x^3$ is $9s = 4a(\sec^2 \psi - 1)$.

3. (a) Evaluate $\int_R \int xy \, dx \, dy$ over the region in the positive quadrant for which $x + y \leq 1$.

   (b) Apply Gauss's divergence theorem to evaluate $[(x^3 - yz)\, dy\, dz - 2x^2y\, dz\, dx + 3dx\, dy]$ over the surface of cube bounded by the co-ordinate planes and the planes x = = z = a.

OR

(a) Evaluate $\int_0^3 \int_0^2 \int_0^1 (x + y + z)\, dx\, dy\, dz$.

(b) Verify Stokes theorem for the functin $\vec{F} = x^2 \hat{i} + xy \hat{j}$ integrated round the square in xy-plane whose sides are along the lines x = 0, y = 0, x = a, y = a.

4. (a) Investigate the continuity of the function :

$$f(x,y)=\begin{cases} \dfrac{xy}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$$

at the point (0, 0).

(b) If $u = (x^2 + y^2 + z^2)^{-1/2}$; $x^2 + y^2 + z^2 \neq 0$ then prove that :

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2} = 0$$

OR

(a) If $u = \sec^{-1}\left[\dfrac{x^8+y^8}{x+y}\right]$ then prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y} = 2 \cot u$.

(b) State and prove mean value theorem for a function of two variable.

5. (a) Discuss the maximum or minimum value of $u = ax^3y^2 - x^4y^2 - x^3y^3$.

(b) Test the convergence of the series :

$$\frac{1}{1.2.3.}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+ \dots\dots\dots.$$

OR

(a) Find the maximum value of u = sin x sin y sin (x + y)

(b). Test the convergence or divergence of the series :

$$\frac{x}{1.2}+\frac{x^2}{3.4}+\frac{x^3}{5.6}+\frac{x^4}{7.8}$$

where x is positive.

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