May 2005

Bachelor of Computer Application (BCA) Examination

II Semester

Mathematics - II

Time 3 Hours] [Max. Marks 40

Note : *All questions are compulsory and carry equal marks. Solve any two parts from each question.*

1. (a) Trace the curve :

$$y^2 (a + x) = x^2(a - x).$$

(b) Test the convergence of the following integrals :

(i) $\int_0^{\infty} \frac{\cos x}{1+x^2} dx$ (ii) $\int_a^{\infty} \frac{e^{-x} \cos x}{x^2} dx$

(c) Trace the curve :

$$r = \frac{1}{2} + \cos 2\theta.$$

2. (a) Prove that :

$$B (m, n) = \frac{\Gamma m \Gamma n}{\sqrt{m+n}} \ (m, n > 0).$$

(b) Show that :

$$\int_0^1 \frac{x^2}{\left(1-x^4\right)^{1/2}} dx \times \int_0^1 \frac{1}{\left(1+x^4\right)^{1/2}} dx = \frac{\pi}{4\sqrt{2}}.$$

(c) Find the length of the arc of the curve :

$$y = \log \left(\frac{e^x - 1}{e^x + 1}\right)$$

from x = 1 to x = 2.

3. (a) Evaluate :

$$\int_1^e \int_0^{\log y} \int_1^{e^x} \log z \, dy \, dx \, dz.$$

(b) If $\vec{r}(t) = 5t^2 \hat{i} + t \hat{j} - t^3 \hat{k}$, then show that :

$$\int_1^2 \vec{r} \times \frac{d^2 \vec{r}}{dt^2} = -14\hat{i} + 75\hat{j} - 15\hat{k}.$$

(c) Apply Gauss's divergence theorem to evaluate $\int\int_S \vec{F} \cdot \hat{n}\, dS$, where $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and S is the surface of the cube bounded by planes x = 0, x = 1, y = 0, y = 1, z = 0, z = 1. 4

4. (a) If $u = e^{xyz}$, then show that :

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}.$$

(b) Show that the function :

$$f(x,y) = \begin{cases} \frac{x^3 - y^3}{x^2 + y^2}, & \text{when}(x,y) \neq (0,0) \\ 0, & \text{when}(x,y) = (0,0) \end{cases}$$

is not differentiable at origin although partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exists at (0, 0).

(c) State and prove Taylor's theorem for a function of two variables.

5. (a) If a, b, c are positive numbers, find the maximum value of $f(x, y, z) = x^a y^b z^c$ subject to the condition x + y + z = 1.

(b) Test the convergence or divergence of the series :

$$x + \frac{3}{5}x^2 + \frac{8}{10}x^3 + \frac{15}{17}x^4 + \ldots + \frac{(n^2-1)}{(n^2+1)}x^n + \ldots \text{ where } x > 0.$$

(c) Discuss the maximum or minimum value of the function :

$$u = xy + \frac{a^3}{x} + \frac{a^3}{y}.$$

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