May-June 2006

Bachelor of Computer Application (BCA) Examination

II Semester

Mathematics - II

Time 3 Hours] [Max. Marks 40

Note : *All questions are compulsory and carry equal marks. Solve any two parts from each qeuestion.*

1. (a) Trace the curve : $x^3 + y^3 = 3axy$.

   (b) Trace the curve : $r^2 = a^2 \cos 2\theta$.

   (c) (i) Define improper integral and explain kinds of improper integrals.

   (ii) Test the convergence of $\int_0^{\pi/4} \frac{1}{\sqrt{\tan x}} dx$.

2. (a) State and prove Legendre's duplication formula.

   (b) Express $\int_0^1 x^m (1 - x^n)^p \, dx$ in terms of the beta function and hence evaluate $\int_0^1 x^5 (1 - x^3)^{10} \, dx$.

   (c) Prove that the whole length of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ is 6a.

3. (a) Prove that :

$$\int\int\int_V x^{e-1} y^{m-1} z^{n-1} \, dx \, dy \, dz = \frac{\Gamma l \Gamma m \Gamma n}{\sqrt{l+m+n+1}}$$

   where V is the closed region bounded by co-ordinate planes and the plane x + y + z = 1.

   (b) If $\vec{r} \times d\vec{r} = \vec{0}$, then show that $\hat{r}$ = constant.

   (c) Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ and C is the rectangle in the xy-plane bounded by x = a, x = 0, y = b, y = 0.

4. (a) If $u = \tan^{-1}\dfrac{xy}{\sqrt{1+x^2+y^2}}$, then prove that $\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{1}{(1+x^2+y^2)^{3/2}}$

(b) State and prove Euler's theorem for a homogeneous function of two variables.

(c) Let $f(x, y) = x^2 - 3xy + 2y^2$. Use mean value theorem to express the difference $f(1, 2) - f(2, -1)$ by partial derivatives. Compute θ and check that it is between 0 and 1.

5. (a) Discuss the maximum or minimum value of $u = x^3y^2(1 - x - y)$

(b) Find the maxima and minima of $u = x^2 + y^2 + z^2$ subject to the conditions $ax^2 + by^2 + cz^2 = 1$ and $lx + my + nz = 0$. Interpret the result geometrically.

(c) Find whether the series :

$$x + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \frac{4^4x^4}{4!} + \ldots\ldots \quad x > 0$$

is convergent or divergent?

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