June - July 2007

Bachelor of Computer Application (BCA) Examination

II Semester

Mathematics - II

Time : 3 Hours] [Max. Marks : 40

Note : *All questions are compulsory and carry equal marks. Solve any two parts from each question.*

1. (a) Trace the curve $a^2y^2=x^2(a^2-x^2)$.

(b) Test the convergence of the integrals :

(i) $\int_0^\infty \frac{\sin^2 x}{x^2}dx$ (ii) $\int_0^{\pi/2} \log \sin x \, dx$.

(c) Trace the curve $r = a(1 - \cos\theta)$.

2. (a) Prove that $B(m,n) = \frac{\Gamma m \, \Gamma n}{\Gamma m+n}$, $(m, n > 0)$.

(b) Evaluate :

(i) $\int_0^\infty \frac{x^8(1-x^6)}{(1+x)^{24}}dx$ (ii) $\int_0^\infty \frac{x^4(1+x^5)}{(1+x)^{15}}dx$.

(c) Prove that the arc length of the curve $y = \log \sec x$ from $x = 0$ to $x = \pi/3$ is $\log_e(2 + \sqrt{3})$.

3. (a) Evaluate $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z} dx\, dy\, dz.$.

(b) Let R be the region between the parabola $y = x^2$ and straight line $y = x + 6$, then evaluate $\int_R \int x \, dA$.

(c) Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ taken round the rectangle bounded by $x = \pm a$, $y = 0$, $y = b$.

4. (a) If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$.

(b) State and prove mean value theorem for a function of two variables.

(c) Investigate the continuity of the function :

$$f(x, y) = \begin{cases} \dfrac{xy^2}{x^2 + y^4} & (x, y) \neq (0,0) \\ 0 & (x, y) = (0,0) \end{cases}$$

5. (a) Discuss the maximum or minimum values of the function

$$u = xy + a^3 \left(\frac{1}{x} + \frac{1}{y}\right)$$

(b) Find the maximum and minimum values of $u = a^2x^2 + b^2y^2 + c^2z^2$ when $x^2 + y^2 + z^2 = 1$ and $lx + my + nz = 0$.

(c) Test the convergence of divergence of series :

$$2x + \frac{3x^2}{8} + \frac{4x^3}{27} + \ldots\ldots + \frac{n+1}{n^3}x^n + \ldots\ldots$$

where x is positive.

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