June - July 2008

Bachelor of Computer Application (BCA) Examination

II Semester

Mathematics - II

Time : 3 Hours] [Max. Marks : 40

Note : *All questions are compulsory and carry equal marks. Solve any two parts from each questions.*

1. (a) Trace the curve $y^2 (a - x) = x^3, a > 0$.
   (b) Test the convergence of the integrals :
   (i) $\int_{-\infty}^{\infty} \frac{dx}{a^2 + x^2}$ (ii) $\int_0^{\pi/2} \tan x \, dx$.
   (c) Trace the curve $r^2 = a^2 \sin 2\theta$.

2. (a) Prove that $\beta (m, n) = \frac{(m-1)(n-1)!}{(m+n-1)!}$, for $m, n > 0$
   (b) Find the length of the spiral $r = e^{\alpha\theta}$ from the pole to the point (r, θ).
   (c) Prove that :
   (i) $\sqrt{\frac{1}{2}} = \sqrt{\pi}$ (ii) $\sqrt{n+1} = n!; n = 1, 2, 3, ...$

3. (a) Evaluate $\int_0^1 \int_0^{\sqrt{(1+x^2)}} \frac{dx\,dy}{1+x^2+y^2}$
   (b) Verify Stoke's theorem for $\bar{A} = y^2 i + xyj - xzk$ where S is the hemisphere $x^2 + y^2 + z^2 = a^2, z > 0$.
   (c) Evaluate $\int \int_S \frac{\bar{r}}{r^2} , \bar{n} \, dS$.

4. (a) Find $\frac{\partial^3 u}{\partial x \partial y \partial z}$ if $u = e^{x^2} + y^2 + z^2$
   (b) Use Taylor's theorem to expand $f(x, y) = x^2 + xy + y^2$ in powers of (x - 1) and (y - 2).
   (c) State and prove Euler's theorem on homogeneous functions.

5. (a) Find the shortest distance from the origin to the curve $xyz^2 = 2$.
   (b) Show that the following series :
   $$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{n(n+1)} + ...$$
   is convergent.
   (c) Test for convergence or divergence of the series :
   $$\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \frac{x^4}{4.5} + ..., x > 0.$$

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