October 2010

Bachelor of Computer Application (BCA) Examination

II Semester

# Mathematics - II

Time : 3 Hours ] [ Max. Marks : 40

***Note :*** *All questions are compulsory and carry equal marks. Solve any two parts from each question.*



1. (a) Trace the curve :

$$Y^2 (a + x) = x^2 (a - x)$$

(b) Trace the polar curve :

$$r = a(1 - \sin\theta)$$

(c) Examine the convergence of the integral :

$$\int_0^1 \frac{dx}{x^{\frac{1}{2}} (1-x)^{\frac{1}{3}}}$$

(d) Show that the integral $\int_a^\infty \frac{\sin^2 x}{x^2} dx$ is convergent.

2. (a) Prove that :

$$\int_0^{\pi/2} \frac{dx}{\sqrt{\sin x}} \times \int_0^{\pi/2} \sqrt{\sin x}\, dx = \pi$$

(b) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ and evaluate $\Gamma\left(-\frac{3}{2}\right)$.

(c) Prove that, the length of the arc of the parabola $y^2 = 4ax$ cut off by the line $3y = 8x$ is $\left[\log 2 + \frac{15}{16}\right]$.

(d) Prove that the intrinsic equation of the parable $3ay^2 = 2x^3$ is $9s = 4a(\sec^2\psi - 1)$



3. (a) Evaluate :

$$\int_1^2 \int_0^{\sqrt{2x-x^2}} x\, dx\, dy$$

(b) Evaluate :

$$\iiint_V z\, dxdydz$$

where the region of integration V is a cylinder which is bounded by $z = 0$, $z = 1$, $x^2 + y^2 = 4$

(c) Changing to polar co-ordinates, evaluate :

$$\int_0^1 \int_0^x \frac{x^3}{\sqrt{x^2 + y^2}} \, dxdy.$$

(d) Show that :

$$\iint_S (axi + byj + czk).ndS = \frac{4}{3} \pi (a + b + c)$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$

4. (a) If $U = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$, then prove that :

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$$

(b) If $f(x, y) = \frac{xy}{\sqrt{x^2 + y^2}}$ $(x, y) \neq (0, 0)$ and show that the function is continuous

$= 0$ $\quad (x, y) \neq (0, 0)$

(c) Expand $f(x, y) = x^2 + xy - y^2$ by Taylor's theorem in the power of $(x - 1)$ and $(y + 2)$.

(d) Let $f(x, y) = xy^2 - x^2y$. Find the proper value of $\theta$ if $a = b = 0$, $h = 1$, $K = 2$ using mean value theorem for two variables.

5. (a) Find the maximum and minimum value of the function :

$$f(x, y) = xy(a - x - y)$$

(b) Find the maxima and minima of $U = x^2 = y^2 + z^2$ where $ax^2 + by^2 + cz^2 = 1$.

(c) Test the convergence of the series :

$$1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \ldots\ldots\ldots + \frac{x^n}{n^2 + 1} + \ldots\ldots\ldots (x > 0)$$

(d) Test for the convergence of the series :

$$x^2 + \frac{2^2}{3.4} x^4 + \frac{2^2.4^2}{3.4.5.6} x^6 + \frac{2^2.4^2.6^2}{3.4.5.6.7.8} x^8 + \ldots\ldots. x > 0$$

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