September 2011

Bachelor of Computer Application (BCA) Examination

II Semester

Mathematics - II

Time : 3 Hours] [Max. Marks : 40

Note : *All questions are compulsory and carry equal marks. Solve any two parts from each question.*

1. (a) Trace the curve $a^2y^2 = x^3(2a - x)$.

   (b) Test the convergence of $\int_0^1 \frac{dx}{x^{1/2}(1-x)^{1/3}}$.

   (c) Define improper integral and explain kinds of improper integral.

2. (a) Prove that $B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$, $m, n > 0$.

   (b) Prove that $\Gamma(n)\,\Gamma(1-n) = \frac{\pi}{\sin n\pi}$, $0 < n < 1$.

   (c) Find the length of spiral $r = e^{a\theta}$ from the pole to the point (r, θ)

3. (a) Evaluate $\int_1^e \int_0^{\log} \int_1^{e^x} \log x \, dy\, dx\, dz$

   (b) Show that $\iint_S (axi + byj + czk).\,n\, dS = \frac{4}{3}\pi(a + b + c)$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$.

   (c) Evaluate $\iint_S \frac{\vec{r}}{r^2}.\bar{n}\; dS$.

4. (a) State and prove Euler's theorem on homogenous functions.

   (b) If $u = e^{xyz}$, then prove that:

   $$\frac{\partial^3 u}{\partial x\, \partial y\, \partial z} = (1 + 3xyz + x^2y^2z^2)\, e^{xyz}.$$

   (c) Expand $f(x,y) = x^2 + xy - y^2$ by Taylor's theorem in the power of $(x - 1)$ and $(y + 2)$.

5. (a) Discuss the maxima and minima of $u = ax^3y^2 - x^4y^2 - x^3y^3$.

   (b) Find the maxima and minima of $u = x^2 + y^2 + z^2$ subject to the conditions $ax^2 + by^2 + cz^2 = 1$ and $lx + my + nz = 0$.

   (c) Show that the following series is convergent :

   $$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots\ldots + \frac{1}{n(n+1)} + \ldots..$$

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