

July 2013

Bachelor of Computer Application (BCA) Examination
II Semester

Mathematics - II

Time : 3 Hours]

[Max. Marks : 40

Note : All questions are compulsory and carry equal marks. Solve any two parts from each question.

1. (a) Trace the curve $y = x^3$ (b) Test the convergence of $\int_0^{\infty} \frac{\cos x}{1+x^2} dx$.(c) Trace the curve $r = a(1 + \cos \theta)$.

2. (a) Prove that :

$$B(m, n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)} \quad (m, n > 0).$$

(b) Show that whole lengths of the curve $x = a \cos^3 t$, $y = a \sin^3 t$ is $6a$.

(c) Prove that :

$$\Gamma n \Gamma(1-n) = \frac{\pi}{\sin n\pi} \quad 0 < n < 1.$$

3. (a) Evaluate :

$$\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) dz dy dx.$$

(b) If $\vec{r}(t) = 5t^2 \mathbf{i} + t\mathbf{j} - t^3 \mathbf{k}$, show that :

$$\int_1^2 \left(\vec{r} \times \frac{d^2 \vec{r}}{dt^2} \right) dt = 14\mathbf{i} + 75\mathbf{j} - 15\mathbf{k}.$$

(c) Show that :

$$\int \int_S (ax\mathbf{i} + by\mathbf{j} + cz\mathbf{k}) \cdot \hat{n} dS = \frac{4}{5} \pi (a + b + c)$$

Where S is the surface of sphere $x^2 + y^2 + z^2 = 1$.

4. (a) If $u = \sin^{-1} \left(\frac{x^2 + y^2}{x + y} \right)$ then show that :

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u.$$

(b) Expand $f(x, y) = x^2y + 3y - 2$ in powers of $(x - 1)$ and $(y + 2)$.

(c) Examine the continuity of the function at $(0, 0)$ where :

$$F(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}} & ; (x, y) \neq (0, 0) \\ 0 & ; (x, y) = (0, 0) \end{cases}$$

5. (a) Show that series $\sum_{n=0}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$ and divergent if

$$p \leq 1.$$

(b) Discuss the maxima and minima of :

$$u = ax^3 y^2 - x^4 y^2 - x^3 y^3$$

(c) Test the convergence of series :

$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots$$

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