July 2014

Bachelor of Computer Application (BCA) Examination

II Semester

Mathematics - II

Time : 3 Hours] [Max. Marks : 40

Note : *All questions are compulsory and carry equal marks. Solve any two parts from each question.*

1. (a) Trance the curve $a^2 y^2 = x^2 (a^2 - x^2)$.

   (b) Trance the curve $r^2 = a^2 \cos 2\theta$.

   (c) Test the convergence of $\int_0^1 \frac{dx}{x^{1/2}(1-x)^{1/3}}$

2. (a) Express $\int_0^1 x^m (1-x^n)^p \ dx$ in terms of the beta function and hence evaluate $\int_0^1 x^5 (1-x^3)^{10} dx$.

   (b) To prove that $\Gamma m \left|\overline{m+\frac{1}{2}}\right. = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma 2m$, where $m > 0$.

   (c) Prove that the arc length of the curve $y = \log \sec x$ from $x = 0$ to $x = \frac{\pi}{3}$ is $\log_e (2 + \sqrt{3})$.

3. (a) Evaluate $\int_0^1 \int_0^{x^2} e^{y/x} \, dy\, dx$.

   (b) Given that :

   $$\vec{r}(t) = 2\hat{i} = \hat{j} + 2\hat{k} \text{ where } t = 2$$

   $$= 4\hat{i} = 2\hat{j} + 3\hat{k} \text{ where } t = 3.$$

   Show that $\int_2^3 \vec{r} . \frac{d\vec{r}}{dt} dt = 10$.

   (c) Verify Gauss's divergence theorem and show that :

   $$\iint_S [(x^3 - yz)\hat{i} - 2x^2 y \hat{j} + 2\hat{k}] . \hat{n} dS = \frac{1}{3} a^5.$$

Where S denotes the surface of the cube bounded by the planes $x = 0,\ x = a,\ y = a,\ z = 0,\ z = a$.

4. (a) If $u = (x^2 + y^2 + z^2)^{-1/2}$; $x^2 + y^2 + z^2 = 0$ then prove that :

(i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=-u$ (ii) $\frac{\partial^2 u}{\partial x^2}+y\frac{\partial^2 u}{\partial y^2}+z\frac{\partial^2 u}{\partial z^2}=0.$

(b) Let $f(x, y) = x^2 - 3xy + 2y^2$. Use mean value theorem to express the difference $f(1, 2) - f(2, -1)$ by Partial derivatives. Compute θ and check that it is between 0 and 1.

(c) Investigate the continuity of the function.

$$f(x,y)=\begin{cases}\dfrac{xy^2}{x^2+y^4}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$ at (0, 0).

5. (a) Discuss the maxima and minima of the function $ax^3y^2 - x^4y^2 - x^3y^3$.

(b) Test the convergence of the following series, where x is positive :

$$x+\frac{3}{5}x^2+\frac{8}{10}x^3+\frac{15}{17}x^4+.....+\frac{n^2-1}{n^2+1}x^n+......$$

(c) Test the convergence of the series :

$$\frac{1}{1^p}+\frac{1}{3^p}+\frac{1}{5^p}+\frac{1}{7^p}+......$$

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