

May 2015

Bachelor of Computer Application (BCA) Examination
II Semester

Mathematics - II

Time : 3 Hours]

[Max. Marks : 40

Note: All questions are compulsory and carry equal marks. Solve any two parts from each question.

1. (a) Trace the curve $y^2 (a - x) = x^2 (a + x)$.

(b) Trace the curve $r = a (1 + \cos \theta)$.

(c) Test the convergence of $\int_0^2 \frac{\log x}{\sqrt{2-x}} dx$.

2. (a) Prove that $B(m, n) = \int_0^x \frac{x^{n-1}}{(1+x)^{m+n}} dx$.

(b) Prove that $\int_0^{\infty} \frac{x^c}{c^x} dx = \frac{\sqrt{c+1}}{(\log c)^{c+1}}$ ($c > 1$).

(c) Find the entire length of the cardioid $r = a (a + \cos \theta)$.

3. (a) Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dx dy}{1+x^2+y^2}$.

(b) If $\vec{r}(t) = 5t^2 \mathbf{i} + t\mathbf{j} - t^3 \mathbf{k}$, show that :

$$\int_1^2 \left(\vec{r} \times \frac{d^2 \vec{r}}{dt^2} \right) dt = -14\mathbf{i} + 75\mathbf{j} - 15\mathbf{k}.$$

(c) Show that $\iint_S (ax\mathbf{i} + by\mathbf{j} + cz\mathbf{k}) \cdot \hat{n} dS = \frac{4}{3} \pi (a+b+c)$.

where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$.

4. (a) If $u = e^{xyz}$, show that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^2) e^{xyz}$

(b) Expand $f(x, y) = x^2 + xy + y^2$ in powers of $(x-2)$ and $(y-3)$ by Taylor's expansion.

davv bca question papers

$$(c) \text{ Let } f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}} & ; (x,y) \neq (0,0) \\ 0 & ; (x,y) = (0,0) \end{cases}$$

Show that $f(x,y)$ is continuous but not differentiable at $(0, 0)$,

5. (a) Discuss the maxima and minima of the function $u(x, y) = x^3y^2(1 - x - y)$.
- (b) Test for convergence of the following series :

$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots$$

- (c) Test the convergence of the series :

$$2x + \frac{3x^2}{8} + \frac{4x^3}{27} + \dots + \frac{(n+1)x^n}{n^3} + \dots, x > 0.$$

□□□