June 2016

Bachelor of Computer Application (BCA) Examination

II Semester

Mathematics - II

Time 3 Hours] [Max. Marks 40

Note : *All questions are compulsory and carry equal marks. Solve any two parts from each questions.*

1. (a) Trace the curve $r^2 = a^2 \sin 2\theta$.

   (b) Trace the curve $y^2 (2a - x) = x^3$.

   (c) Test the convergence of $\int_0^1 \frac{dx}{x^{1/2}(1-x)^{1/3}}$.

2. (a) Prove that $\beta(m, n) \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$

   (b) Show that $\int_0^2 x(8-x^3)^{1/3} dx = \frac{16\pi}{9\sqrt{3}}$.

   (c) Find the length of the arc of the curve $y = \log \frac{e^x - 1}{e^x + 1}$ from $x = 0$ to $x = \pi/3$.

3. (a) Evaluate $\int_0^3 \int_0^2 \int_0^1 (x + y + z)\, dx\, dy\, dx$.

   (b) Evaluate $\int_C F.dr$ where $F = (x^2 + y^2)\, i - 2xy\, j$ and the curve C is $y^2 = 4x$ in the xy plane bounded by $x = 0$, $x = a$, $y = b$, $y = 0$.

   (c) If $\vec{r} \times d\vec{r} = \vec{0}$ then show that $\hat{r}$ = constant.

4. (a) if $u = \log (x^3 + y^3 + z^3 - 3xyz)$ then prove that:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}.$$

   (b) State and prove Mean Value Theorem for functions of two variables.

   (c) Show that the function:

$$f(x,y) = \begin{cases} \frac{xy^2 + x^2y}{x^3 + y^3} & ; \quad (x,y) \neq (0,0) \\ 0 & ; \quad (x,y) = (0,0) \end{cases}$$

   is discontinuous at the origin (0, 0)

5. (a) Discuss the maxima and minima of function:

$$u(x, y) = xy + \frac{a^3}{x} + \frac{a^3}{y}.$$

(b) Find the minimum value of $u = x^2 + y^2 + z^2$ of having given $ax + by + cz = P$.

(c) Test the convergence of the series:

$$\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \frac{x^4}{4.5} + \ldots\ldots \text{ where } x > 0.$$

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